ILLINOIS CORE MATH STANDARDS (8-4-10)

The Common Core Standards (CCS) for K-12 mathematics have recently been adopted by Illinois and several other states. The developmental mathematics curriculum at the community college level historically has reflected the mathematics content taught at the high school level. In light of the proposed changes to K-12 mathematics curricula, it is necessary to examine the developmental mathematics curricula in Illinois. The Illinois Mathematics and Computer Science Articulation Guide as adopted by IMACC and ISMAA describes an optimal structure for developmental mathematics in Illinois.

The charge of this committee is to examine the developmental mathematics curriculum as defined in the Joint Articulation Guide with respect to recently adopted core standards for K-12 mathematics.

The Standards for Mathematical Practice, as presented in the CCS Standards, are listed below, followed by excerpts from the Articulation Guide, addressing the content of the specific CCS Standard.

1. Make sense of problems and persevere in solving them.

The Articulation Guide reflects this goal by promoting the following:

- active involvement of students in solving real multi-step mathematics problems.
- introduction of needed skills in the context of real applications.
- emphasizing mental arithmetic, estimation, and the translation of problem situations into algebraic models.
- integration of mathematical topics so that students may use a wide range of mathematical content and techniques to solve problems.

2. Reason abstractly and quantitatively.

The Articulation Guide on page 5 supports this goal, as noted from the following excerpt.

Problem solving and logical reasoning should be a main thread throughout all developmental-remedial courses. This does not imply the elimination of all skill development and algebraic manipulation that currently dominate the developmental-remedial curriculum; however, there needs to be adequate time for students to develop conceptual and mathematical understanding.

In addition, the Articulation Guide states,

- "Deductive reasoning should be an integral part of the [geometry] course."

3. Construct viable arguments and critique the reasoning of others.

The Articulation Guide on page 15 recognizes that one of the goals of the developmental mathematics curriculum is to emphasize:

- conceptual understanding of mathematical ideas and the ability to use valid arguments
- integration of interactive learning involving collaborative groups.

4. Model with mathematics.

The Articulation Guide on page 6 demonstrates the importance of modeling, as seen in the following statement regarding quantitative literacy.

The mathematics component of general education focuses on quantitative reasoning to provide a base for developing a quantitatively literate college graduate. Every college graduate should be able to apply simple mathematical methods to the solution of real-world problems. A quantitatively literate college graduate should be able to:

- interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them;
- represent mathematical information symbolically, visually, numerically, and verbally;
- use arithmetic, algebraic, geometric, and statistical methods to solve problems;
- estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results;
- recognize the limitations of mathematical and statistical models.

5. Use appropriate tools strategically.

An excerpt from page 4 of the Articulation Guide states:

Technology Statement
The appropriate use of technology is an essential part of many mathematics courses. Effective and strategic usage of technology by both students and faculty is highly encouraged. As is stated in the Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus (AMATYC, 1995, p.12), Technology should be used to enhance the study of mathematics but should not become the main focus of instruction. The amount of time that students spend learning how to use computers and calculators effectively must be compatible with the expected gain in learning mathematics. Computer software, especially packages appropriate for demonstration or visual representation of mathematical concepts, is strongly recommended. The use of calculators in any pre-algebra level course is best determined by departmental philosophy at the local level.

In particular, on pages 14/15, the Articulation Guide addresses the issue of technology in developmental mathematics by stating:

It is highly recommended that increased attention be given to:

- the appropriate use of technology throughout the curriculum for computational work, graphing, and geometry.

6. Attend to precision.

The Articulation Guide does not emphasize the importance of precision. However, it is partially reflected in the following quotation from page 21, in the course description for Geometry.

<u>Although emphasis should be placed on techniques and manipulations, problem solving and logical reasoning should be a main thread throughout the course</u>. Much effort should be given to utilize instruction that will provide students with needed techniques and also enable students to reason and make the connections that are involved in the learning of mathematics. The instruction should emphasize the connections between verbal, numerical, symbolic and graphical representations of the concepts being taught wherever possible.

7. Look for and make use of structure.

The Articulation Guide, on pages 14/15 includes the following statement:

 It is highly recommended that increased attention be given to:

- the application of multiple approaches (numerical, graphical, symbolic, and verbal) to help the students learn a variety of techniques for solving problems (Cohen, 1995, p. 29).

8. Look for and express regularity in repeated reasoning.

As noted in item 6 above,

- The instruction should emphasize the connections between verbal, numerical, symbolic and graphical

However, this is one area that could be strengthened in the Articulation Guide. IMACC/ISMAA may want to consider a statement that encourages students to develop new rules by extending ideas.

Core Standards Topics for Pre-algebra

Fractions
    Definition of a fraction
    Equivalent fractions
    Decimals
    Ordering fractions and decimals
    Addition and Subtraction, Multiplication and Division
    Complex fractions
    Percent, ratio, and rate

Geometry
    Basic vocabulary (polygons, angle, parallelism, perpendicularity
    Length, area, and volume (*informal* definitions and basic formulas)

Measurements (?)
    English and metric systems

Rational numbers
    The two-sided number line
    Addition, and subtraction as addition
    Multiplication, and division as multiplication
    Absolute value

Real numbers
    Fundamental Assumption of School Mathematics
    Commutative, Associative and distributive properties of addition and multiplication
    The Inverse relationship between addition and subtraction
    The Inverse relationship between multiplication and division
    Laws of exponents for rational (and real?) exponents

Illinois Learning Objectives for Arithmetic

1. Perform arithmetic operations with real numbers–whole numbers, integers, fractions, decimals and signed numbers.

2. Calculate and/or solve percentages, ratios, and proportions.

3. Convert within and use the English and metric measurement systems.

4. Use basic geometric terminology and formulas, such as perimeter, area and volume.

5. Use the above topics in routine applications.

6. Recognize the reasonableness of solutions.

Illinois Learning Objectives for Pre-algebra

1. Perform arithmetic operations with integers, rational numbers (fractional, decimal, and mixed number forms), real numbers, algebraic expressions and polynomials.

2. Solve linear equations and inequalities in one variable.

3. Solve and graph linear equations in two variables.

4. Apply the laws of exponents.

5. Apply geometric concepts of perimeter, area, and volume.

6. Demonstrate the basic concepts of roots and applications of roots.

7. Find the least common multiple using the prime factorization method.

8. Convert between fractional, decimal, and percent forms and apply these concepts to basic percent problems.

9. Apply the order of operations to numerical and algebraic expressions.

10. Apply the Pythagorean Theorem.

11. Use algebra to solve applications.

12. Solve real world problems involving measurement, percent, fractions, decimals and square roots.

Comments: The Core Standards Pre-algebra topics are present in the learning objectives for arithmetic and pre-algebra. The Illinois standards include solving equations, graphing equations and Pythagorean theorem.

Core Standards Topics for Basic Algebra

Use of symbols
  Expressions
  Definition of a variable
  Equations and inequalities

Coordinate systems in the plane
  Meaning of coordinates
  Graphs of equations

Similar triangles
  Definition and the fundamental theorem
  Informal discussions of SAS, AA, SSS
  Applications

Linear equations

Meaning of *solving* an equation
Word problems
Definition of slope of a line
Why graph of linear equations are lines
Finding equation of a given line
Characterization of parallel lines in terms of slope
Characterization of perpendicular lines in terms of slope
Simultaneous linear equations

Illinois Learning Objectives for Basic Algebra

1. Use the terms, definitions, and notation of basic algebra.

2. Identify and make use of real number properties and evaluate real number expressions.

3. Sketch the graph of a linear function and identify slope and intercepts.

4. Perform operations with polynomials.

5. Solve linear and quadratic equations.

6. Solve application problems and then recognize the reasonableness of solutions.

7. Apply laws of exponents.

Comments: The Core Standards for Basic Algebra are reflected in the combined Illinois Learning Objectives for Pre-Algebra and Basic Algebra. The Illinois Standards include quadratic equations and polynomials. The Core Standards include properties of triangles.

Question: Should Illinois consider including similar triangles on the list of topics for pre-algebra or basic algebra?

Core Standards Topics for Intermediate Algebra

Functions

Graphs of functions; relationship with graphs of equations
Linear functions of one variable and inequalities
Quadratic functions
Graphs of quadratic functions
Quadratic Equations and the quadratic formula

Polynomials (= polynomial functions)

Basic theorems
Complex numbers
Solutions of complex quadratic polynomials
Statement of Fundamental Theorem of Algebra

Roots of real polynomials
Rational functions

Exponential functions
Rational exponents
Exponential functions and their graphs
Concept of inverse functions
Relationship between graphs of inverse functions
Logarithms and their graphs
Basic theorems

Sequences
Generate an arithmetic or geometric sequence given a recursive rule for the sequence

Modeling
Linear, quadratic and Exponential models

Illinois Learning Standards for Intermediate Algebra

1. Perform arithmetic operations with real numbers, complex numbers, and algebraic expressions including polynomials, rational expressions, and radical expressions.

2. Solve linear, rational, radical, absolute value, *logarithmic and *exponential equations in one and two variables with application of domain and range.

3. Solve linear inequalities and compound inequalities in one and two variables.

4. Factor polynomials, including binomials and trinomials, and identify prime polynomials.

5. Use various methods to solve quadratic equations, including the quadratic formula.

6. Write equations of lines and determine if lines are parallel or perpendicular.

7. Use graphs to identify solutions to linear equations and inequalities in one and two variables, as well as systems of equations and inequalities in two variables.

8. Solve systems of linear equations in two and three variables.

9. *Apply laws of logarithms and exponents to simplify logarithmic and exponential expressions and to solve equations and applications.

10. Graph quadratic, *exponential, and *logarithmic functions.

11. Solve applications involving linear expressions, equations and inequalities, rational equations, radical equations, and systems of equations.

12. Identify and solve applications involving direct, inverse and/or joint variation.

Comments: Most of the topics from the Core Standards list are present on the Illinois list. Illinois is missing the topic of sequences. The Core Standards do not seem to be emphasizing factoring as much as Illinois. In addition, the Core Standards emphasize modeling.

Question: Should the Illinois learning objectives for intermediate algebra include sequences and a greater emphasis on modeling?

Core Standards Topics for Geometry

Congruence of Rectilinear Figures
> Basic Figures
> Rigid Motions
> Theorems about Lines and Angles
> Congruence of Triangles
> Theorems about Triangles
> Quadrilaterals

Similarity of Rectilinear Figures
> Dilations
> Fundamental Theorem of Similarity
> Similarity of Triangles
> Pythagorean Theorem and its converse

Circles

Basic Constructions

(Downplay logic and axioms and Give proofs)

Illinois Learning Objectives for Geometry

1. Use the concepts of undefined terms, definitions, postulates, and theorems in the logical development of geometry.

2. Perform constructions using a straightedge and compass and/or computer generated constructions.

3. Use inductive reasoning to form conjectures.

4. Write proofs using deductive (including indirect) reasoning.

5. Apply theorems of congruency to prove triangles and parts of triangles congruent.

6. Solve applications related to parallel and perpendicular lines.

7. Solve applications related to parallelograms, regular polygons, and other polygons.

8. Use similarity to solve applications.

9. Use the Pythagorean Theorem to solve applications.

10. Solve applications involving circles.

11. Apply formulas to solve problems related to perimeter, area, and volume.

Comments: The topic lists appear similar however one could argue that there are substantial differences in perspectives. The Core Standards seem to emphasize similarity and measurement while the Illinois topics seem to emphasize logic and axioms.

Question: Should the Illinois Learning Standards for geometry better reflect the philosophy of the Core Standards?

8-4-10